# ELECTROMAGNETISMO I - FORMULARIO

## Tema 1.1 Álgebra vectorial

| | | |
|---|---|---|
| $\overline{A} = A\hat{a} \quad \hat{a} = \frac{\overline{A}}{A}$ | $r = \lvert r \rvert = \left(r_x^2 + r_y^2 + r_z^2\right)^{\frac{1}{2}}$ | $r = x\hat{x} + y\hat{y} + z\hat{z} \ \ (m)$ |

Cosenos directores

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|---|---|---|---|
| $l_x = \cos\alpha = \frac{A_x}{A}$ | $l_y = \cos\beta = \frac{A_y}{A}$ | $l_z = \cos\gamma = \frac{A_z}{A}$ | $\hat{\boldsymbol{a}} = l_x\hat{\boldsymbol{x}} + l_y\hat{\boldsymbol{y}} + l_z\hat{\boldsymbol{z}}$<br>$l_x^2 + l_y^2 + l_z^2 = 1$ |

## Tema 1.2. Cálculo diferencial

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|---|---|---|
| $d\boldsymbol{r} = dx\hat{\boldsymbol{x}} + dy\hat{\boldsymbol{y}} + dz\hat{\boldsymbol{z}}$ | $\nabla u = \frac{\partial u}{\partial x}\hat{\boldsymbol{x}} + \frac{\partial u}{\partial y}\hat{\boldsymbol{y}} + \frac{\partial u}{\partial z}\hat{\boldsymbol{z}}$ | $\nabla \cdot \boldsymbol{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$ |
| $\nabla \times \boldsymbol{A} = \begin{vmatrix} \hat{\boldsymbol{x}} & \hat{\boldsymbol{y}} & \hat{\boldsymbol{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$ | $\nabla^2 u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$ | $\nabla^2 \boldsymbol{A} = (\nabla^2 A_x)\hat{\boldsymbol{x}} + (\nabla^2 A_y)\hat{\boldsymbol{y}} + (\nabla^2 A_z)\hat{\boldsymbol{z}}$ |
| $\nabla \times \nabla u = \boldsymbol{0}$ | $\nabla \cdot (\nabla \times \boldsymbol{A}) = 0$ | $\nabla(uv) = u\nabla v + v\nabla u$ |
| $\nabla \cdot (u\boldsymbol{A}) = \boldsymbol{A} \cdot (\nabla u) + u(\nabla \cdot \boldsymbol{A})$ | $\nabla \times (\nabla \times \boldsymbol{A}) = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A}$ | $\nabla \times (u\boldsymbol{A}) = (\nabla u) \times \boldsymbol{A} + u(\nabla \times \boldsymbol{A})$ |

## Tema 1.3 Teoremas de la divergencia y el rotacional

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|---|---|
| $\oint_S \boldsymbol{A} \cdot d\boldsymbol{a} = \int_V (\nabla \cdot \boldsymbol{A}) d\tau$ | $\oint_L \boldsymbol{A} \cdot d\boldsymbol{l} = \int_S (\nabla \times \boldsymbol{A}) d\boldsymbol{a}$ |

## Tema 1.4 Delta de Dirac

En 1 dimensión:

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|---|---|---|
| $\int_{-\infty}^{+\infty} f(x)\delta(x-a)\,dx = f(a)$ | $\delta(kx) = \frac{1}{\lvert k \rvert}\delta(x)$ | $\delta(-x) = \delta(x)$ |

En 3 dimensiones:

| | |
|---|---|
| $\delta^3(\boldsymbol{r}-\boldsymbol{r}') = \delta(x-x')\delta(y-y')\delta(z-z') = \begin{cases} 0, & \boldsymbol{r} \neq \boldsymbol{r}' \\ \infty, & \boldsymbol{r} = \boldsymbol{r}' \end{cases}$ | $\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} f(\boldsymbol{r})\delta^3(\boldsymbol{r}-\boldsymbol{r}')dxdydz = f(\boldsymbol{r}')$ |
| $\boldsymbol{\nabla} \cdot \left(\frac{\hat{\boldsymbol{R}}}{R^2}\right) = 4\pi\delta^3(\boldsymbol{R})$ ; $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$ | $\nabla^2\left(\frac{1}{R}\right) = -4\pi\delta^3(\boldsymbol{R})$ |
| $\rho(\boldsymbol{r}) = q'\delta^3(\boldsymbol{r}-\boldsymbol{a}) - q'\delta^3(\boldsymbol{r})$ | $\rho(\boldsymbol{r}) = A\delta(r-R)$ |

## Tema 2.1 Ley de Coulomb

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|---|---|---|---|
| $\nabla \cdot \boldsymbol{E} = \rho/\varepsilon_0$ | $\nabla \cdot \boldsymbol{B} = 0$ | $\nabla \times \boldsymbol{E} = -\partial \boldsymbol{B}/\partial t$ | $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 (\partial \boldsymbol{E}/\partial t)$ |
| | | $\nabla \times \boldsymbol{E} = \boldsymbol{0}$ | $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$ |

| | | |
|---|---|---|
| $\boldsymbol{F}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q'q}{R^2}\hat{\boldsymbol{R}} = \frac{1}{4\pi\varepsilon_0}\frac{q'q}{R^3}\boldsymbol{R}$ | $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}' \rightarrow R = \lvert \boldsymbol{r} - \boldsymbol{r}' \rvert$ | $\boldsymbol{F}(\boldsymbol{r}) = \frac{q}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\boldsymbol{r}')d\tau'\hat{\boldsymbol{R}}}{R^2}$ |
| $\boldsymbol{F}(\boldsymbol{r}) = \boldsymbol{F}_{q_1' \rightarrow q} + \boldsymbol{F}_{q_2' \rightarrow q} + \cdots = \sum_{i=1}^{n} \boldsymbol{F}_{q_i' \rightarrow q} = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0}\frac{q_i' q}{R_i^3}\boldsymbol{R}_i$ | | |

## Tema 2.2 Campo E, flujo de E, ley de Gauss, y condiciones de contorno

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|---|---|---|
| $\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R^2}\hat{\boldsymbol{R}} = \frac{1}{4\pi\varepsilon_0}\frac{q'}{R^3}\boldsymbol{R}$ | $\boldsymbol{E}(\boldsymbol{r}) = \sum_{i=1}^{n}\boldsymbol{E}_i(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{R_i^2}\hat{\boldsymbol{R}}$ | $\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\boldsymbol{r}')d\tau'\hat{\boldsymbol{R}}}{R^2}$ |
| $\nabla \cdot \boldsymbol{E} = \frac{\rho(\boldsymbol{r}')}{\varepsilon_0}$ $F(r) = qE(r)$ | $\oint_S \boldsymbol{E} \cdot d\boldsymbol{a} = \frac{Q_{enc}}{\varepsilon_0}$ | $E_2^{\perp} - E_1^{\perp} = (\boldsymbol{E}_2 - \boldsymbol{E}_1)\hat{\boldsymbol{n}} = \frac{\sigma}{\varepsilon_0}$ $E_2^{\parallel} = E_1^{\parallel}$ |

## Tema 2.3 Potencial electroestático

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|---|---|---|
| $V(\boldsymbol{r}) = \sum_{i=1}^{n}\frac{q_i}{4\pi\varepsilon_0 R_i}$ | $V(\boldsymbol{r}) = \frac{1}{4\pi\varepsilon_0}\int_{V'}\frac{\rho(\boldsymbol{r}')d\tau'}{R}$ | $V(\boldsymbol{r}) = \sum_{i=1}^{n} V_i(\boldsymbol{r})$ |
| $\Delta V = V(\boldsymbol{r}_b) - V(\boldsymbol{r}_a) = -\int_a^b \boldsymbol{E}(\boldsymbol{r})d\boldsymbol{r}$ | $\nabla^2 V = -\frac{\rho(\boldsymbol{r})}{\varepsilon_0}$ | $\nabla^2 V = 0$ $\boldsymbol{E}(\boldsymbol{r}) = -\nabla V(\boldsymbol{r})$ |

## Tema 2.4 Electroestática en el vacío y en medios materiales

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|---|---|---|
| $U_e = qV(\boldsymbol{r})$ | ó | $U_e = \frac{\varepsilon_0}{2}\int_{\text{todo el espacio}} E^2\, d\tau$ |

| | | |
|---|---|---|
| | $U_e = \frac{1}{2}\sum_{i=1}^{n} q_i V(\boldsymbol{r}_i)$ | |
| $U_e = \frac{1}{2}\int_{L'} \lambda(\boldsymbol{r}')\, V(\boldsymbol{r}')dl'$ | $U_e = \frac{1}{2}\int_{V'} \rho(\boldsymbol{r}')\, V(\boldsymbol{r}')d\tau'$ | $u_e = \frac{1}{2}\varepsilon_0 E^2$ |

## Tema 2.5 Conductores en campos electroestáticos

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|---|---|---|
| $C = \frac{Q}{\Delta V}$ | $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$ | $C_{eq} = C_1 + C_2$ |

### Coordenadas cilíndricas

Vector en cilíndricas:

$$\boldsymbol{A} = A_s\hat{\boldsymbol{s}} + A_\varphi\hat{\boldsymbol{\varphi}} + A_z\hat{\boldsymbol{z}}$$

$$\boldsymbol{r} = s\hat{\boldsymbol{s}} + z\hat{\boldsymbol{z}}$$

**Cilíndricas a cartesianas:**

$$\begin{aligned} x &= s\cos\varphi \\ y &= s\sin\varphi \\ z &= z \end{aligned}$$

**Cartesianas a cilíndricas:**

$$\begin{aligned} s &= (x^2 + y^2)^{1/2} \\ \varphi &= \arctan(y/x) \\ z &= z \end{aligned}$$

En cuanto a los vectores unitarios:

$$\begin{aligned} \hat{\boldsymbol{x}} &= \cos\varphi\,\hat{\boldsymbol{s}} - \sin\varphi\,\hat{\boldsymbol{\varphi}} \\ \hat{\boldsymbol{y}} &= \sin\varphi\,\hat{\boldsymbol{s}} + \cos\varphi\,\hat{\boldsymbol{\varphi}} \\ \hat{\boldsymbol{z}} &= \hat{\boldsymbol{z}} \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{s}} &= \cos\varphi\,\hat{\boldsymbol{x}} + \sin\varphi\,\hat{\boldsymbol{y}} \\ \hat{\boldsymbol{\varphi}} &= -\sin\varphi\,\hat{\boldsymbol{x}} + \cos\varphi\,\hat{\boldsymbol{y}} \\ \hat{\boldsymbol{z}} &= \hat{\boldsymbol{z}} \end{aligned}$$

$$\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{s}} = \hat{\boldsymbol{\varphi}}\cdot\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{z}} = 1$$
$$\hat{\boldsymbol{s}}\cdot\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{\varphi}}\cdot\hat{\boldsymbol{z}} = \hat{\boldsymbol{z}}\cdot\hat{\boldsymbol{s}} = 0$$
$$\hat{\boldsymbol{s}}\times\hat{\boldsymbol{\varphi}} = \hat{\boldsymbol{z}} \quad \hat{\boldsymbol{\varphi}}\times\hat{\boldsymbol{z}} = \hat{\boldsymbol{s}} \quad \hat{\boldsymbol{z}}\times\hat{\boldsymbol{s}} = \hat{\boldsymbol{\varphi}}$$

$$\frac{d\hat{\boldsymbol{s}}}{d\varphi} = \hat{\boldsymbol{\varphi}}$$

$$\frac{d\hat{\boldsymbol{\varphi}}}{d\varphi} = -\hat{\boldsymbol{s}}$$

Diferencial del vector desplazamiento:

$$d\boldsymbol{r} = ds\hat{\boldsymbol{s}} + sd\varphi\hat{\boldsymbol{\varphi}} + dz\hat{\boldsymbol{z}}$$

Diferenciales de área:
Diferencial de volumen:

$$\text{Si } s = cte \rightarrow da_s = \pm s d\varphi dz$$
$$\text{Si } \varphi = cte \rightarrow da_\varphi = \pm ds dz$$
$$\text{Si } z = cte \rightarrow da_z = \pm s ds d\varphi$$

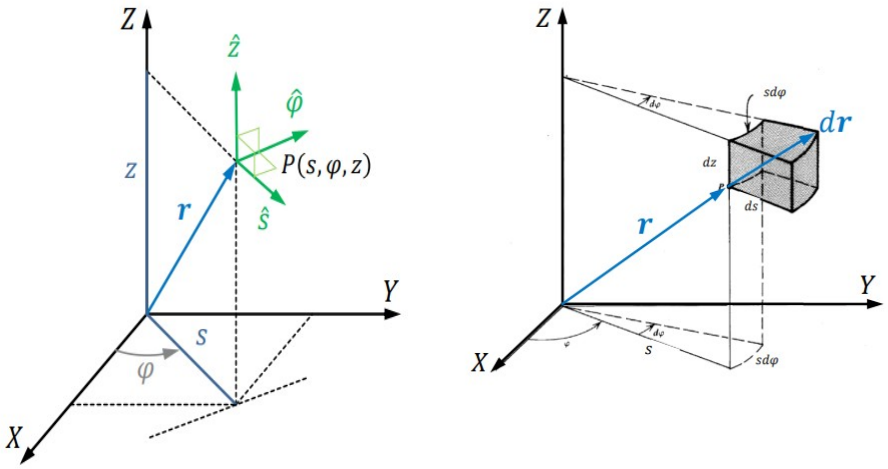
$$d\tau = s ds d\varphi dz$$

**Gradiente:** $\nabla u(s,\varphi,z) = \frac{\partial u}{\partial s}\hat{\boldsymbol{s}} + \frac{1}{s}\frac{\partial u}{\partial \varphi}\hat{\boldsymbol{\varphi}} + \frac{\partial u}{\partial z}\hat{\boldsymbol{z}}$

**Divergencia:** $\nabla \cdot \boldsymbol{A}(s,\varphi,z) = \frac{1}{s}\frac{\partial (sA_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$

**Rotacional:** $\nabla \times \boldsymbol{A}(s,\varphi,z) = \left(\frac{1}{s}\frac{\partial A_z}{\partial \varphi} - \frac{\partial A_\varphi}{\partial z}\right)\hat{\boldsymbol{s}} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s}\right)\hat{\boldsymbol{\varphi}} + \frac{1}{s}\left[\frac{\partial (sA_\varphi)}{\partial s} - \frac{\partial A_s}{\partial \varphi}\right]\hat{\boldsymbol{z}}$

**Laplaciano:** $\nabla^2 u(s,\varphi,z) = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial u}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 u}{\partial \varphi^2} + \frac{\partial^2 u}{\partial z^2}$




## Coordenadas esféricas

Vector en esféricas:

$$\boldsymbol{A} = A_r\hat{\boldsymbol{r}} + A_\theta\hat{\boldsymbol{\theta}} + A_\omega\hat{\boldsymbol{\varphi}}$$

$$\boldsymbol{r} = r\hat{\boldsymbol{r}}$$

**Esféricas a cartesianas:**

$$\begin{aligned} x &= r\sin\theta\cos\varphi \\ y &= r\sin\theta\sin\varphi \\ z &= r\cos\theta \end{aligned}$$

**Cartesianas a esféricas:**

$$\begin{aligned} r &= (x^2+y^2+z^2)^{1/2} \\ \tan\theta &= (x^2+y^2)^{1/2}/z \\ \tan\varphi &= y/x \end{aligned}$$

En cuanto a los vectores unitarios:

$$\begin{aligned} \hat{\boldsymbol{r}} &= \sin\theta\cos\varphi\,\hat{\boldsymbol{x}} + \sin\theta\sin\varphi\,\hat{\boldsymbol{y}} + \cos\theta\,\hat{\boldsymbol{z}} \\ \hat{\boldsymbol{\theta}} &= \cos\theta\cos\varphi\,\hat{\boldsymbol{x}} + \cos\theta\sin\varphi\,\hat{\boldsymbol{y}} - \sin\theta\,\hat{\boldsymbol{z}} \\ \hat{\boldsymbol{\varphi}} &= -\sin\varphi\,\hat{\boldsymbol{x}} + \cos\varphi\,\hat{\boldsymbol{y}} \end{aligned}$$

$$\begin{aligned} \hat{\boldsymbol{x}} &= \sin\theta\cos\varphi\,\hat{\boldsymbol{r}} + \cos\theta\cos\varphi\,\hat{\boldsymbol{\theta}} - \sin\varphi\,\hat{\boldsymbol{\varphi}} \\ \hat{\boldsymbol{y}} &= \sin\theta\sin\varphi\,\hat{\boldsymbol{r}} + \cos\theta\sin\varphi\,\hat{\boldsymbol{\theta}} + \cos\varphi\,\hat{\boldsymbol{\varphi}} \\ \hat{\boldsymbol{z}} &= \cos\theta\,\hat{\boldsymbol{r}} - \sin\theta\,\hat{\boldsymbol{\theta}} \end{aligned}$$

$$\frac{d\hat{\boldsymbol{r}}}{d\theta} = \hat{\boldsymbol{\theta}} \qquad \frac{d\hat{\boldsymbol{r}}}{d\varphi} = \sin\theta\,\hat{\boldsymbol{\varphi}} \qquad \frac{d\hat{\boldsymbol{\theta}}}{d\theta} = -\hat{\boldsymbol{r}} \qquad \frac{d\hat{\boldsymbol{\theta}}}{d\varphi} = \cos\theta\,\hat{\boldsymbol{\varphi}} \qquad \frac{d\hat{\boldsymbol{\varphi}}}{d\theta} = 0 \qquad \frac{d\hat{\boldsymbol{\varphi}}}{d\varphi} = -\sin\theta\,\hat{\boldsymbol{r}} - \cos\theta\,\hat{\boldsymbol{\theta}}$$

Diferencial del vector desplazamiento:

$$d\boldsymbol{r} = dr\hat{\boldsymbol{r}} + rd\theta\hat{\boldsymbol{\theta}} + r\sin\theta\,d\varphi\hat{\boldsymbol{\varphi}}$$

Diferenciales de área: Diferencial de volumen:

$$\begin{aligned} &\text{Si } r = cte \rightarrow da_r = \pm r^2\sin\theta\,d\theta d\varphi \\ &\text{Si } \theta = cte \rightarrow da_\theta = \pm r\sin\theta\,dr d\varphi \\ &\text{Si } \varphi = cte \rightarrow da_\varphi = \pm r dr d\theta \end{aligned}$$

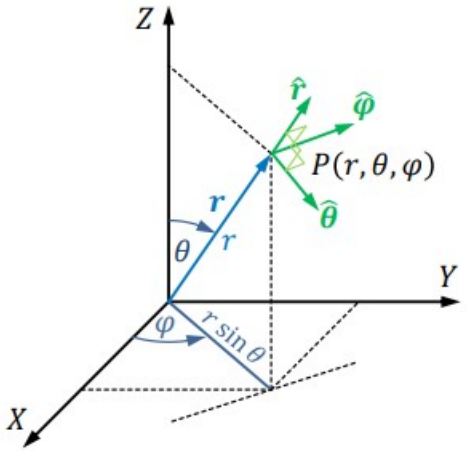
$$d\tau = r^2\sin\theta\,dr d\theta d\varphi$$

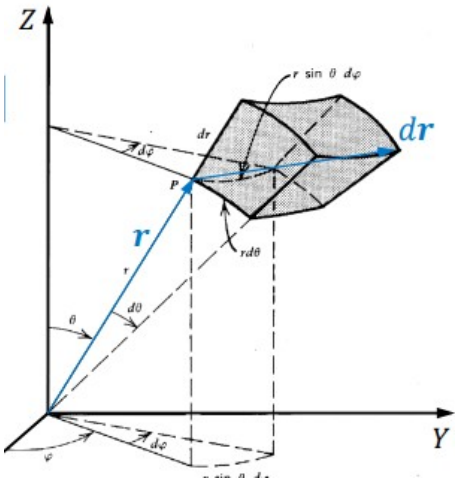
**Gradiente:** $\nabla u(r,\theta,\varphi) = \frac{\partial u}{\partial r}\hat{\boldsymbol{r}} + \frac{1}{r}\frac{\partial u}{\partial\theta}\hat{\boldsymbol{\theta}} + \frac{1}{r\sin\theta}\frac{\partial u}{\partial\varphi}\hat{\boldsymbol{\varphi}}$

**Divergencia:** $\nabla\cdot\boldsymbol{A}(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2A_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(\sin\theta\,A_\theta) + \frac{1}{r\sin\theta}\frac{\partial A_\varphi}{\partial\varphi}$

**Rotacional:** $\nabla\times\boldsymbol{A}(r,\theta,\varphi) = \frac{\hat{\boldsymbol{r}}}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta\,A_\varphi) - \frac{\partial A_\theta}{\partial\varphi}\right] + \frac{\hat{\boldsymbol{\theta}}}{r}\left[\frac{1}{\sin\theta}\frac{\partial A_r}{\partial\varphi} - \frac{\partial}{\partial r}(rA_\varphi)\right] + \frac{\hat{\boldsymbol{\varphi}}}{r}\left[\frac{\partial}{\partial r}(rA_\theta) - \frac{\partial A_r}{\partial\theta}\right]$

**Laplaciano:** $\nabla^2u(r,\theta,\varphi) = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial u}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial u}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2u}{\partial\varphi^2}$


Z
Y
X
$\hat{r}$
$\hat{\varphi}$
$\hat{\theta}$
$P(r,\theta,\varphi)$
r
r
θ
φ
r sin θ


Z
Y
r sin θ dφ
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